P4-2, page 204: Future value calculation

Use the basic formula for future value along with the given interest rate, \( i \), and the number of periods, \( n \), to calculate the future value interest factor, \( FVIF \), in each of the cases shown in the following table.

<table>
<thead>
<tr>
<th>Case</th>
<th>Interest rate, ( i )</th>
<th>Number of periods, ( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12%</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>6%</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>9%</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>3%</td>
<td>4</td>
</tr>
</tbody>
</table>

Basic formula for \( FVIF_{i,n} \): \( FVIF_{i,n} = (1+i)^n \)

**Calculation procedure:**

\[
\begin{align*}
A & \quad FVIF_{12\%,2} = (1+0,12)^2 = 1,2544 \\
B & \quad FVIF_{6\%,3} = (1+0,06)^3 = 1,1910 \\
C & \quad FVIF_{9\%,2} = (1+0,09)^2 = 1,1881 \\
D & \quad FVIF_{3\%,4} = (1+0,03)^4 = 1,1255
\end{align*}
\]

P4-3, page 204: Number of periods estimation

Use the \( FVIF \) in Appendix Table A-1 in each of the cases shown in the table on the facing page to estimate, to the nearest year, how long it would take an initial deposit, assuming no withdrawals, a) to double, b) to quadruple.

<table>
<thead>
<tr>
<th>Case</th>
<th>Interest rate, ( i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>7%</td>
</tr>
<tr>
<td>B</td>
<td>40%</td>
</tr>
<tr>
<td>C</td>
<td>20%</td>
</tr>
<tr>
<td>D</td>
<td>10%</td>
</tr>
</tbody>
</table>

Basic formula for \( FV_n \): \( FV_n = PV \times (1+i)^n \) or \( FV_n = PV \times FVIF_{i,n} \)

a) To double the initial deposit. It means \( FV=2, PV=1 \)
b) To quadruple the initial deposit. It means \( FV=4, PV=1 \)
**Calculation procedure using FVIF table:**

**A**

a) We have to find for which FVIF is the number of periods, n, nearest to 2:

\[ 2 = \text{FVIF}_{7\%},n \]

We find in the FVIF table that \text{FVIF}_{7\%},11=2,105 and \text{FVIF}_{7\%},10=1,967.

So the number of periods is between 10 and 11 years. Number of periods, n, is nearest 10 years in this case.

b) We have to find for which FVIF is the number of periods, n, nearest to 4

\[ 4 = \text{FVIF}_{7\%},n \]

Find the nearest number of periods in the FVIF table in column for 7%. n is between 20 and 21 years.

**B**

a) \[ 2 = \text{FVIF}_{40\%},n, \quad 2 \text{ years} < n < 3 \text{ years}, \text{nearest to 2 years} \]

b) \[ 4 = \text{FVIF}_{40\%},n, \quad 4 \text{ years} < n < 5 \text{ years}, \text{nearest to 4 years} \]

**C**

a) \[ 2 = \text{FVIF}_{20\%},n, \quad 3 \text{ years} < n < 4 \text{ years}, \text{nearest to 4 years} \]

b) \[ 4 = \text{FVIF}_{20\%},n, \quad 7 \text{ years} < n < 8 \text{ years}, \text{nearest to 8 years} \]

**D**

a) \[ 2 = \text{FVIF}_{10\%},n, \quad 7 \text{ years} < n < 8 \text{ years}, \text{nearest to 7 years} \]

b) \[ 4 = \text{FVIF}_{10\%},n, \quad 14 \text{ years} < n < 15 \text{ years}, \text{nearest to 15 years} \]

**Another calculation procedure using logarithm:**

**A**

a) \[ 2= \text{FVIF}_{7\%},n=1,07^n \quad \text{/log} \]

\[ \log2/\log1,07=n \]

\[ n=10,2448 \text{ years} \]

b) \[ 4=\text{FVIF}_{7\%},n=1,07^n \quad \text{/log} \]

\[ \log4/\log1,07=n \]

\[ n=20,49 \text{ years} \]

**B**

a) \[ 2=\text{FVIF}_{40\%},n=1,40^n \quad \text{/log} \]

\[ \log2/\log1,40=n \]

\[ n=2,06 \text{ years} \]
b) \[ 4 = FVIF_{40\%} \cdot n = 1,40^n \quad /\log \]
\[ \log 4 / \log 1,40 = n \]
\[ n = 4,12 \text{ years} \]

C a) \[ 2 = FVIF_{20\%} \cdot n = 1,20^n \quad /\log \]
\[ \log 2 / \log 1,20 = n \]
\[ n = 3,8 \text{ years} \]

b) \[ 4 = FVIF_{20\%} \cdot n = 1,20^n \quad /\log \]
\[ \log 4 / \log 1,20 = n \]
\[ n = 7,6 \text{ years} \]

D a) \[ 2 = FVIF_{10\%} \cdot n = 1,10^n \quad /\log \]
\[ \log 2 / \log 1,10 = n \]
\[ n = 7,25 \text{ years} \]

b) \[ 4 = FVIF_{10\%} \cdot n = 1,10^n \quad /\log \]
\[ \log 4 / \log 1,10 = n \]
\[ n = 14,55 \text{ years} \]
P4-4, page 205: Future value calculation

For each of the cases shown in the following table, calculate the future value of the single cash flow deposited today that will be available at the end of the deposit period if the interest is compounded annually at the rate specified over the given period.

<table>
<thead>
<tr>
<th>Case</th>
<th>Single CF, PV</th>
<th>Interest rate, i</th>
<th>Deposit period, n</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>200</td>
<td>5%</td>
<td>20</td>
</tr>
<tr>
<td>B</td>
<td>4 500</td>
<td>8%</td>
<td>7</td>
</tr>
<tr>
<td>C</td>
<td>10 000</td>
<td>9%</td>
<td>10</td>
</tr>
<tr>
<td>D</td>
<td>25 000</td>
<td>10%</td>
<td>12</td>
</tr>
<tr>
<td>E</td>
<td>37 000</td>
<td>11%</td>
<td>5</td>
</tr>
<tr>
<td>F</td>
<td>40 000</td>
<td>12%</td>
<td>9</td>
</tr>
</tbody>
</table>

Calculation procedure using FVIF table: $FV_n = PV \times FVIF_{i,n}$

<table>
<thead>
<tr>
<th>Case</th>
<th>$FV_n$</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$FV_n = 200 \times FVIF_{5%,20}$</td>
<td>$200 \times 2,653 = 530,6\text{ USD}$</td>
</tr>
<tr>
<td>B</td>
<td>$FV_n = 4500 \times FVIF_{8%,7}$</td>
<td>$4500 \times 1,714 = 7,713\text{ USD}$</td>
</tr>
<tr>
<td>C</td>
<td>$FV_n = 10 000 \times FVIF_{9%,10}$</td>
<td>$10 000 \times 2,367 = 23,670\text{ USD}$</td>
</tr>
<tr>
<td>D</td>
<td>$FV_n = 25 000 \times FVIF_{10%,12}$</td>
<td>$25 000 \times 3,138 = 78,450\text{ USD}$</td>
</tr>
<tr>
<td>E</td>
<td>$FV_n = 37 000 \times FVIF_{11%,5}$</td>
<td>$37 000 \times 1,685 = 62,345\text{ USD}$</td>
</tr>
<tr>
<td>F</td>
<td>$FV_n = 40 000 \times FVIF_{12%,9}$</td>
<td>$40 000 \times 2,773 = 110,920\text{ USD}$</td>
</tr>
</tbody>
</table>
P4-8, page 206: Interest rate determination

Misty needs to have 15 000 USD at the end of 5 years to fulfill her goal of purchasing a small sailboat. She is willing to invest the funds as a single amount today but wonders what sort of investment return she will need to earn.

a) She can invest 10 200 USD today
b) She can invest 8 150 USD today
c) She can invest 7 150 USD today

**Calculation procedure:** \( FV_n = PV \times FVIF_{i,n} \)

We have to find interest rate which results into 15 000USD at the end of 5 years.

a) \( FV = 15 000, PV = 10 200, n = 5, i = ? \)

\[
15 000 = 10 200 \times FVIF_{1.5}
\]
\[
FVIF_{1.5} = \frac{15 000}{10 200}
\]
\[
FVIF_{1.5} = 1.4706 \quad 8\% < i < 9\%
\]

If we want to find exact interest rate value, we have to use linear interpolation:

- Calculate or find in the table FVIF for lower and higher interest rate:

  \[
  FVIF_{8\%,5} = 1.4693
  \]
  \[
  FVIF_{1\%,5} = 1.4706
  \]
  \[
  FVIF_{9\%,5} = 1.5386
  \]

  - Linear interpolation

  \[
  i = 8 + \frac{(1.4706 - 1.4693)}{(1.5383 - 1.4693)} \times (9 - 8) = 8.02\%
  \]

b) \( FV = 15 000, PV = 8 150, n = 5, i = ? \)

\[
15 000 = 8 150 \times FVIF_{1.5}
\]
\[
FVIF_{1.5} = \frac{15 000}{8 150}
\]
\[
FVIF_{1.5} = 1.840 \quad 12\% < i < 13\%
\]

\[
FVIF_{12\%,5} = 1.7623
\]
\[
FVIF_{13\%,5} = 1.8424
\]

\[
 i = 12 + \frac{(1.840 - 1.7623)}{(1.8424 - 1.7623)} \times (13 - 12) = 12.97\%
\]
c) \( FV = 15000, PV = 7150, n = 5, i = ? \)

\[
15000 = 7150 \times FVIF_{i,5}
\]

\[
FVIF_{i,5} = \frac{15000}{7150}
\]

\[
FVIF_{i,5} = 2.098
\]

15\% < i < 16\%

\[
FVIF_{15\%,5} = 2.0114
\]

\[
FVIF_{16\%,5} = 2.1003
\]

\[ i = 15 + \frac{(2.098 - 2.0114)}{(2.1003 - 2.0114)} \times (16 - 15) = 15.974\% \]

P4-16, page 207: Present value and comparisons of two single amounts

In exchange for a 20000 USD payment today, a well-known company will allow you to choose one of the alternatives shown in the following table. Your opportunity cost is 11\%.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Single amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>28500 USD at the end of 3 years</td>
</tr>
<tr>
<td>B</td>
<td>54000 USD at the end of 9 years</td>
</tr>
<tr>
<td>C</td>
<td>160000 USD at the end of 20 years</td>
</tr>
</tbody>
</table>

a) Find the value today of each alternative.
b) Are all the alternatives acceptable?
c) Which alternative, if any, will you take?

**Calculation procedure: \( PV = FV_n \times PVIF_{in} \)**

a) A \( PV = 28500 \times PVIF_{11\%,3} = 20833.5 \) USD (20838.95 USD using calculator)

B \( PV = 54000 \times PVIF_{11\%,9} = 21114 \) USD (21109.94 USD)

C \( PV = 160000 \times PVIF_{11\%,20} = 19840 \) USD (19845.43 USD)

b) Only alternatives A and B exceed 20000 USD.

c) Alternative B because its PV is the greatest and also greater than 20000 USD offer.
Hal Thomas, a 25-year-old college graduate, wishes to retire at age 65. To supplement other sources of retirement income, he can deposit 2,000 USD each year into a tax-deferred individual retirement arrangement assumed to be attainable over the next 40 years.

a) If Hal makes annual end-of-year 2,000 USD deposit into the IRA, how much will he have accumulated by the end of his sixty-fifth year?
b) If Hal decides to wait until age 35 to begin making annual end-of-year 2,000 USD deposits into the IRA, how much will he have accumulated by the end of his sixty-fifth year?
c) Using your findings in parts a) and b), discuss the impact of delaying making deposits into the IRA for 10 years on the amount accumulated by the end of Hal’s sixty-fifth year.
d) Rework parts a), b), c) assuming that Hal makes all deposits at the beginning, rather than the end, of each year. Discuss the effect of beginning-of-year deposits on the future value accumulated by the end of Hal’s sixty-fifth year.

Calculation procedure: $FV_{an}=PMT*FVIFA_{i,n}$

a) $PMT=2,000, \ i=10\%, \ n=40$

$FVA_{40}=2,000*FVIFA_{10\%,40}=885,160$ USD ($885,185.11$ USD)

b) $PMT=2,000, \ i=10\%, \ n=30$

$FVA_{30}=2,000*FVIFA_{10\%,30}=328,982$ USD ($328,988.05$ USD)

c) $FVA_{40}-FVA_{30}=556,178$ USD

10-years delaying the deposits results in more than half a million USD difference between annuities future values. This difference is due to the lost deposits of 20,000 USD and the lost compounding of interest on all of the money for 10 years.

d) All deposits are made at the beginning of each year, it brings additional one year interest:

$FVA_{40}=2,000*FVIFA_{10\%,40}*1.1=973,676$ USD ($973,706.62$ USD)

$FVA_{30}=2,000*FVIFA_{10\%,30}*1.1=361,880$ USD ($361,886.85$ USD)

Both deposits increased due to the extra of compounding from the beginning-of-year deposits instead of the end-of-year deposits.
P4-27, page 210: FV of a mixed stream

For each of the mixed streams of CFs shown in the following table, determine the future value at the end of the final year if deposits are made into an account paying annual interest of 12%, assuming that no withdrawals are made during the period and that the deposits are made:

a) At the end of each year
b) At the beginning of each year

<table>
<thead>
<tr>
<th>Year</th>
<th>CF stream, PMT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>900</td>
</tr>
<tr>
<td>2</td>
<td>1000</td>
</tr>
<tr>
<td>3</td>
<td>1200</td>
</tr>
</tbody>
</table>

Calculation procedure:

a)

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of years to compound, n</th>
<th>FV=PMT*FVIF_{12%},n</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>900* FVIF_{12%},2</td>
<td>1129,0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1000* FVIF_{12%},1</td>
<td>1120,0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1200</td>
<td>1200,0</td>
</tr>
</tbody>
</table>

Value of mixed stream: 3449,0

b) Payments are made at the beginning of each period- value of mixed stream from a) has to be multiplied by (1+i):

\[ 3 449,0 \times 1,12 = 3 862,9 \]